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Sarah Phillips

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Membership Probabilities of White Dwarfs in Open Star Clusters

An Honors Thesis

Sarah Phillips

Directed by Dr. Kurtis Williams

Associate Professor

Department of Physics and Astronomy

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Sarah Phillips

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Membership Probabilities of White Dwarfs in Open Star Clusters

Abstract

This thesis explores the relationship between astronomical position data and star cluster membership. Our research project focuses on the mathematical probability of cluster membership for white dwarf stars (WDs), specifically membership candidates within the open cluster Messier 35. Through Gaussian analysis of proper motion data, we examine the membership probability of each candidate. Establishing cluster membership has broad applications and can allow further determination of a star’s characteristics including stellar age and evolution. Of particular interest are two WD member candidates for Messier 35: LAWDS 4, a DB WD with a helium-dominated atmosphere as opposed to the more common hydrogen-dominated atmospheres, and LAWDS 28, a hot DQ white dwarf with a carbon-dominated atmosphere of uncertain origin. Through the application of a two-dimensional Gaussian model to existing proper motion data for LAWDS 4, LAWDS 28, and other potential WD Messier 35 members, this thesis evaluates their respective membership probabilities and examines potential areas of interest for further research. Various existing literature is also referenced that reinforces our approach to membership probability calculation.

Introduction

Astronomy is a widely varied field of study, with many objects to observe across the night sky. Within this vast ocean of research opportunities, there is a class of objects known as white dwarf (WD) stars. These stars are fascinating to astronomers due to their high densities and remarkable faintness, making them challenging to study (Bergeron et al. 2019). The mass of a WD is typically 0.6 solar mass (sixty percent of the mass of Earth's Sun) but ranges upward to an extraordinary 1.4 solar mass (Chandrasekhar 1931, 1935).

Interestingly, these massive stars typically occupy a volume similar to Earth; this means they contain a large amount of mass within a small volume, making them one of the universe's most dense objects (Althaus et al. 2018). The formation of WDs is relatively common throughout the universe. However, it is challenging to study their life cycles, formation, and material composition in detail; they are some of the most intrinsically faint objects in the sky, making them difficult to observe. Additionally, we rarely possess information regarding the mass, total age, and metal content of progenitors for field WDs, which are not associated with any particular star cluster.

This research project focuses on data analysis concerning WDs that originate from stars that are 1.0 solar mass or greater. Determining whether a WD is a member of a star cluster is a helpful tool that can provide crucial data concerning its position, movement patterns, material composition, and evolution (Liebert et al. 2005). Open star cluster members share several crucial characteristics, including age, metallicity, and movement patterns through space. Establishing that a WD is a cluster member reveals these data, which can help us determine the initial-final mass relation, the relationship between a WDs mass and the mass of its progenitor star (Weidemann 1977, 2000). While many studies have focused on the initial-final mass relation for open cluster WDs, in many instances, the cluster membership of the WDs in their respective clusters is assumed rather than proven (e.g., Claver et al. 2001; Williams et al. 2009; Cummings et al. 2018).

Our research consists of analyzing several massive WDs' positional and proper motion data to determine whether they are members of the star clusters in whose fields they appear. By approaching our data from a mathematical perspective and conducting probability-based membership calculations, we provide accurate estimations of the cluster membership of our WD targets. The targets we analyze from Dr. Williams's previous research will concern stars of interest that are too faint to be accurately measured by the *Gaia* satellite, meaning they are on the visible band with a magnitude fainter than 21 (V > 21). Our analysis of these data provides insight concerning the positions of our WDs within their star clusters. By estimating membership likelihood for these WDs, we gain new insights concerning their compositions, ages, and evolutionary patterns.

During Dr. Williams' observational work conducted over the past several years, he has collected multiple epochs of photometric data on various star clusters. These data guided our focus to several specific WD cluster candidates within Messier 35. One of these objects of interest is LAWDS 28. This star is known as a "hot DQ" WD, meaning that its spectra are highly populated with atomic and ionized carbon absorption lines (Liebert et al. 2003). This atomic carbon concentration in the opacity of a WD is relatively rare; around 80% of WDs have hydrogen-rich atmospheres, with most of the remaining stars exhibiting helium-dominated atmospheres (Williams et al. 2006). Messier 35 is known to be a young star cluster, and the presence of a hot DQ WD within it would challenge the widely held hypothesis that hot DQ WDs form solely via the merging of two normal WDs (Dunlap & Clemens 2015). Since the merger of two WDs is thought to be a process that takes billions of years, the formation process of LAWDS 28 would have taken longer than the age of the star cluster (which, according to Williams et al. 2009 is thought to be roughly 150 million years). By demonstrating that LAWDS

28 is a member of Messier 35, we would be building a case for alternate formation methods of hot DQ WDs.

Another star of interest in Messier 35 is LAWDS 4. This star is a DB WD, meaning its atmosphere is helium-dominated as opposed to the more common hydrogen-dominated DA WDs (Bédard et al. 2020). New studies of DB WDs have led to the hypothesis that these stars may have lower masses than hydrogen-rich WDs, even though they form from the same progenitor star mass (Barnett et al. 2021). However, more data is needed to test this hypothesis, and the evaluation of new DBs will provide helpful insights. Demonstrating the cluster membership of LAWDS 4 would give us valuable information on the comparative masses of DB WDs and their DA counterparts (Williams et al. 2009).

Literature Review

Larger-mass WDs can form in two ways. The first method involves the death of a massive red giant star. The red giant's core, surrounded by a shell of highly-pressurized hydrogen, contracts as the hydrogen fuses and causes the giant's exterior layers to expand outwards (Sparks & Stecher 1974). After the cessation of nuclear fusion, the outer layers of the star's gases form a planetary nebula that surrounds a newly-born WD (Williams et al. 2009). The second method of forming a massive WD is the simple merging of two lower-mass WDs, around 0.6 solar masses each, which happen to orbit one another (Saio et al. 1985).

WDs are particularly fascinating to astronomers because most stars end their lives as WDs; our Sun is among the 97% of stars in our universe that will eventually become WDs (Bédard et al. 2020). The cooling sequence of WDs has been extensively studied, providing a large amount of observational data concerning their aging processes (Debes et al. 2011). In many cases, this makes it possible for us to evaluate the age of a WD based on its temperature and current cooling rate. Identifying the age of WDs allows us to determine the ages of various astronomical structures, such as open star clusters (e.g., von Hippel 2005, Jeffery et al. 2016), globular clusters (e.g., Hansen et al. 2004, 2007), and even components of the Galaxy itself (e.g., Winget et al. 1987, Wood 1992, Kilic et al. 2017). Althaus et al. (2018) describe WDs as “independent reliable cosmic clocks,” thanks to their cooling sequence, with which we can estimate the ages of a vast array of stellar objects.

Another important reason to study WDs is the overwhelming impact they have on chemical processes within the Galaxy. As a star transitions into a WD, it sheds its outer layers of carbon, oxygen, and nitrogen. At this stage of their evolution, WD progenitors appear in the upper right portion of the Hertzsprung–Russell diagram in a section known as the asymptotic giant branch (AGB). This branch houses stars that are incredibly luminous and emit a large amount of energy as they shed their outer layers of material (Barnett et al. 2021). Membership in the AGB branch evidences the intense chemical changes occurring within WD progenitors; these stars lose an incredible amount of mass as they transform, providing the perfect opportunity to study the extraordinary chemistry involved in galactic evolution.

A beneficial tool when studying WDs is the star cluster phenomenon. Star clusters are a gravitationally-bound collection of stars born simultaneously from the same materials. These clusters are circular and tend to be denser than the surrounding areas in space. As a result of their synchronous birth, every member of a star cluster has approximately the same proper motion, or apparent movement through the sky (e.g., Prišegen et al. 2021). Star clusters can provide vital information about WDs and our Galaxy; analysis of star clusters offers information on the metallicity in the Milky Way, its evolution, and even the origins of our Galaxy (Balaguer-Núñez et al. 2020). Furthermore, evidence shows that almost all stars are gravitationally bound to a cluster at some point in their lifetimes, and star clusters provide the chance to study objects in this state (Lada 2010).

A complication arises when studying star clusters due to their placement in the sky. Open clusters are embedded in the disk of our Galaxy, often resulting in partial obscuration by the ubiquitous clouds of gas and dust in the Galactic disk. As a result, physical observations do not always provide an accurate representation of which stars are genuine cluster members. If a field star (a non-cluster-member) happens to pass between an observer and the cluster, or even if it drifts inside the cluster, it could be incorrectly labeled as a cluster member (Richer et al. 2021).

The solution to this potential inaccuracy is to evaluate whether a star is a cluster member based on measurements of the star’s distance (parallax) and motion through space (proper motion). Since cluster members have nearly identical proper motion and parallax, it becomes feasible to calculate the probability of cluster membership based on a star’s motion data (Jones and Prosser 1996). This membership probability calculation is critical for studying WDs and other star cluster members, yet it is often ignored in WD literature (see, e.g., Bellini et al. 2010). It is also often difficult to ascertain where the actual cluster portion ends and the outer regions of a cluster begin. This makes statistics-based calculations of the boundaries necessary, as these calculations allow us to factor in numerous potential errors (Romanishin & Angel 1980).

Methodology

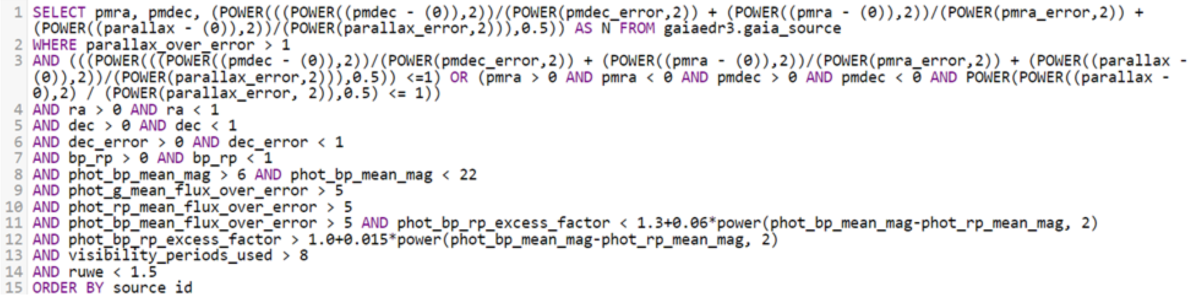
Our goal in this project was to determine cluster memberships of WDs using proper motions. We began our research by analyzing proper motion data collected by the *Gaia* satellite. *Gaia*, which is operated by the European Space Agency, collects a wide variety of astronomical data. These data have been compiled into a stellar catalog available for public use online, which is updated with periodic data releases. The most recent release during our introductory research was the *Gaia* Early Data Release 3 (EDR3), which was made public in December 2020.

The *Gaia* data are considered the most up-to-date, accurate measurements available for many cluster member WDs (Richer et al. 2021). *Gaia* offers extensive proper motion and parallax measurements, which are invaluable to the astronomical research community; when combined with existing photometric data, they allow astronomers to analyze star clusters more accurately. The information collected by the *Gaia* satellite is crucial in studying both already-discovered star clusters and exploring potential members of new ones; therefore, it was the perfect data catalog to use when building and testing our probability calculation process.

In order to validate our methodology, we began by reproducing the results of other astronomers who have already utilized similar methodology. Most notably, we reproduced the results of a paper written in 2020 that utilized data from *Gaia* Data Release 2 (see Prišegen et al. 2020) to search for previously unknown WD members of nearby clusters and associations.

Within the *Gaia* database, we used a form of Structured Query Language (SQL) known as Astronomical Data Query Language (ADQL) to query the database for our required measurements. The statement we constructed (see **Fig. 1**) narrowed the returned results to contain only stars that fit enough cluster member criteria to be considered cluster candidates. The parameters constrain results to stars that match an acceptable range of values for declination, right ascension, proper motion in declination, proper motion in right ascension, and parallax.

These values described the movement patterns and locations of the stars we wanted to analyze, ensuring that we narrowed our data to a manageable size for analysis.



```
SELECT pmra, pmdec, (POWER(((POWER((pmdec - (0)),2))/(POWER(pmdec_error,2)) + (POWER((pmra - (0)),2))/(POWER(pmra_error,2)) +
(POWER((parallax - (0)),2))/(POWER(parallax_error,2))),0.5)) AS N FROM gaiaedr3.gaia_source
WHERE parallax_over_error > 1
AND (((POWER(((POWER((pmdec - (0)),2))/(POWER(pmdec_error,2)) + (POWER((pmra - (0)),2))/(POWER(pmra_error,2)) + (POWER((parallax -
(0)),2))/(POWER(parallax_error,2))),0.5)) <=1) OR (pmra > 0 AND pmra < 0 AND pmdec > 0 AND pmdec < 0 AND POWER(POWER((parallax -
0),2) / (POWER(parallax_error, 2)),0.5) <= 1))
AND ra > 0 AND ra < 1
AND dec > 0 AND dec < 1
AND dec_error > 0 AND dec_error < 1
AND bp_rp > 0 AND bp_rp < 1
AND phot_bp_mean_mag > 6 AND phot_bp_mean_mag < 22
AND phot_g_mean_flux_over_error > 5
AND phot_rp_mean_flux_over_error > 5
AND phot_bp_mean_flux_over_error > 5 AND phot_bp_rp_excess_factor < 1.3+0.06*power(phot_bp_mean_mag-phot_rp_mean_mag, 2)
AND phot_bp_rp_excess_factor > 1.0+0.015*power(phot_bp_mean_mag-phot_rp_mean_mag, 2)
AND visibility_periods_used > 8
AND ruwe < 1.5
ORDER BY source id
```

Fig. 1. A sample ADQL query showing the model we used for our searches. The expected values for proper motion in declination/right ascension and parallax of a cluster member are inserted into the query, along with previously measured error expectations, right ascension, declination, desired visibility period, and several additional photometric parameters. The search result is a data table of stars that meet the requested criteria, along with their Gaia measurements and the parameter Nσ for each star (defined in text).

We also constructed a unique search parameter, which we referred to as N_σ; this parameter allowed us to perform sigma clipping on the data (see **Fig. 2**). N_σ is comparable to the number of standard deviations from the mean cluster values in a WD's two-dimensional proper motion and one-dimensional parallax. Since error values in the *Gaia* data are not fully characterized, we could not formally use the Gaussian probability distribution to determine membership probability; instead, N_σ served as a functional version of a normal distribution for our analysis, representing the statistical significance of cluster membership.

$$N\sigma = \left[\frac{(\mu_\sigma - \mu_\sigma')^2}{\Delta(\mu_{\sigma\varepsilon})^2} + \frac{(\mu_\alpha - \mu_\alpha')^2}{\Delta(\mu_{\alpha\varepsilon})^2} + \frac{(\pi - \pi')^2}{\Delta(\pi_\varepsilon)^2} \right]^{1/2}$$

Fig. 2. The N_σ parameter used for error reduction within our ADQL query was based on the Normal (Gaussian) Distribution; here, it is expressed in mathematical notation. The symbols are defined as follows: μ_σ for proper motion in declination, μ_α for proper motion in right ascension, and π for parallax. The ε subscript on each value in the denominator denotes the estimated error in that value.

After querying the database with our ADQL statement, we automatically exported the resulting measurements of each cluster candidate in the form of a comma-separated values (CSV) file. This file could then be viewed and edited via Excel or any similar data software. We chose to utilize Excel when formatting the data for convenience when editing data headers or re-sorting value columns.

Once we obtained the necessary data for each potential cluster member, we further analyzed these stars using traditional data plotting methods. Using Python, we coded two similar computer programs that can export the measurements from a CSV file into digital scatter plots. The code utilizes Matplotlib, a Python library, to render data directly from a CSV file into customizable graphs. These programs were specifically designed to produce two particular kinds of graphs, the proper motion diagram and the color magnitude diagram (CMD), each of which are particularly useful when visually analyzing star clusters. The proper motion vector-point diagram plots the proper motion in right ascension and proper motion in declination relative to one another; this establishes a star's apparent motion across the sky[1]. The CMD plots the magnitude of a star against its color, which allows us to visually classify stars, providing important insight as we evaluate cluster membership.

Previously Prišegen et al. (2020) utilized the second *Gaia* data release (GDR2) in their analysis, so we applied our ADQL query statement to GDR2 and GEDR3 and compared the results. In doing so, we determined that there were no significant differences in the measurements; we encountered some minor changes in error estimations, but these changes were

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[1] For reference, this process is similar to plotting a point on a two-dimensional geographic map where lines of latitude run horizontally across the page and lines of longitude run vertically; right ascension would be the astronomical equivalent of longitude, and declination would be latitude.

so minuscule as to have minimal effect on our findings. Therefore, although we decided to use the more recent GDR3 as the catalog edition for our queries, the result was a nearly identical reproduction of the Prišegen et al. (2020) results (see **Fig. 3**).

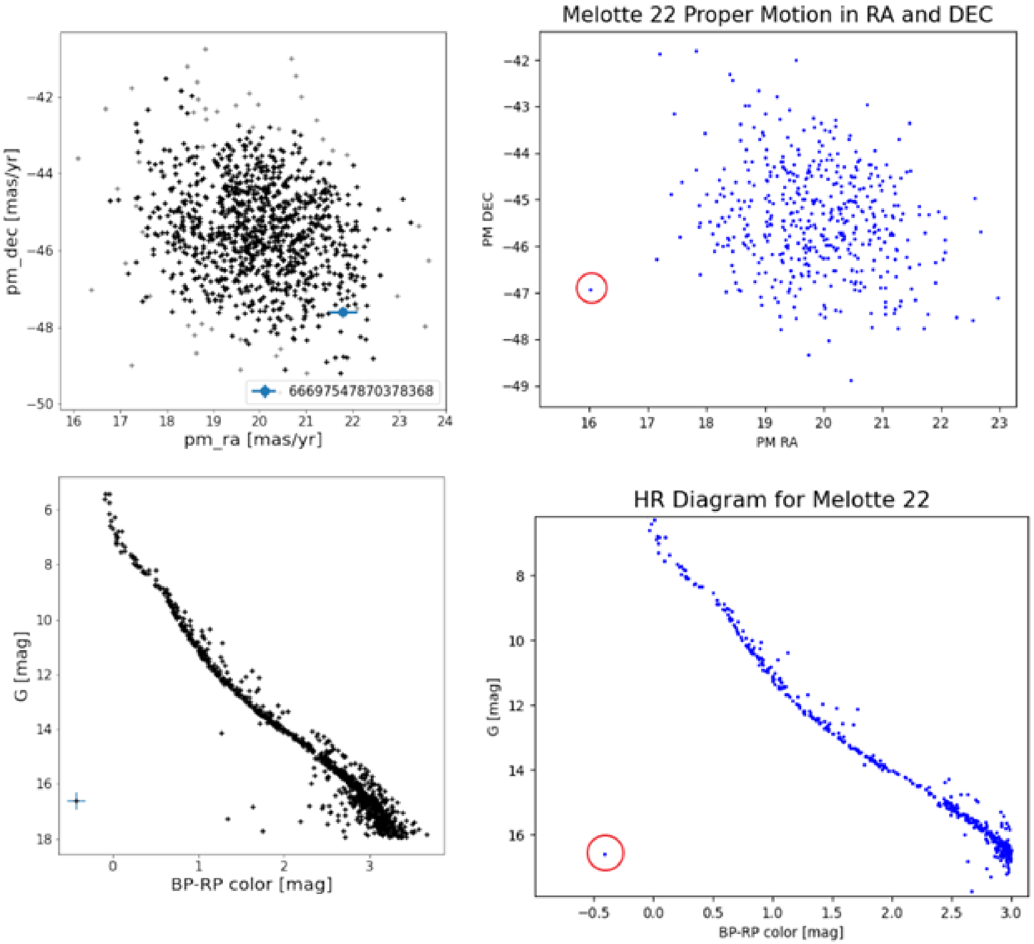


Fig. 3. A comparison of the proper motion diagrams (top) and CMDs (bottom) of Melotte 22 as seen in Prisegen et al. (2020; left) versus our findings (right). Both analyses recovered the established WD cluster candidate EGGR 25 (GAIADR3 2566697547870378368), marked by the blue cross in the Prisegen et al. images and the red circle in ours. The updated EGDR3 data used in our analyses contained more accurate error estimation, resulting in slightly cleaner graphs.

By reproducing the findings of other researchers and searching the newest dataset for possible members, we established a standard for how we would conduct our probability calculations on non-*Gaia* data during our thesis project. Building the ADQL query statement allowed us to pinpoint which parameters would be necessary for our probability analysis. Furthermore, the Python programs used in graphing CMDs and proper motion diagrams for the

Gaia data proved helpful for our thesis project; the code could be easily modified to accept input in the form of any data table and produce the graphs we needed in our analysis.

The welcome discovery of an existing catalog containing highly accurate proper motion measurements of many Messier 35 members, including our potential WD candidates, proved beneficial during our research process. This catalog, which is publicly available online through the Strasbourg Astronomical Data Center, is a previously analyzed collection of data captured during the *Kepler* K2 mission (Bouy et al. 2015). The collection stems from a Dynamical Analysis of Nearby Clusters (DANCe) study of Messier 35, which combines archival and emerging photometric data in an attempt to establish guidelines for cluster membership (Buoy et al. 2015). After reviewing the proper motion data within the catalog and comparing it to Dr. Williams' existing research, we determined that utilizing this collection would be more expedient and accurate than independently generating proper motions from our own photometric data.

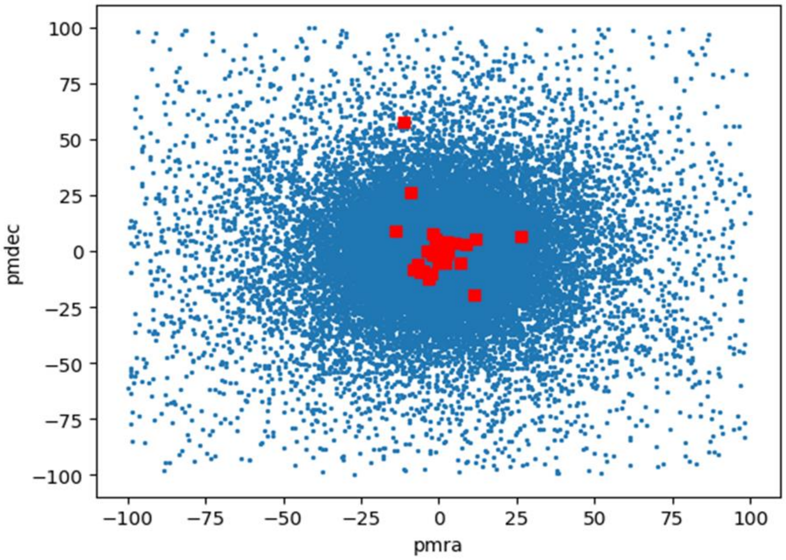


Fig. 4. A scatterplot of the proper motions of all stars in the DANCe Messier 35 catalog (Buoy et el. 2015). WD member candidates are plotted as red squares in comparison to the other stars in the catalog (plotted in blue). Through close examination of the catalog data, we determined that it would be expedient and effective to use the DANCe measurements in our probability calculations.

After determining that we would use the DANCe catalog proper motions, we developed a probability equation for our analysis. While DANCe calculated their own membership probabilities, these estimates included an intrinsic assumption that candidate stars were located on the main sequence of the cluster. Since WDs are not main sequence stars, this methodology was invalid for our work. We therefore used a version of an existing proper motion probability algorithm that does not include the stellar evolutionary state as a prior (Jones & Prosser 1996).

We excluded the portion of this algorithm that considered the distance of the star from the cluster center, as the WD candidates were chosen from imaging of the cluster core only, and no recent cluster density profile was found in the literature concerning Messier 35. Using the Gaussian function as our main reference, we included mean and standard deviation parameters for both cluster and field star proper motions in right ascension and declination. We calculated the projected number density of cluster stars as a function of the radius from cluster center and used this value to ascertain field and cluster densities. The resulting equation was two-dimensional and calculated the membership probability of a star based on existing cluster data. We built out the equation in Python, making it easy to use and modify as needed.

We first established the reliability of our equation by inputting proper motion data for known Messier 35 cluster members. The resulting probability values aligned with our expectations and the previous probability values measured by the DANCe survey team (see **Fig. 5**). After we had concluded that the equation worked as expected for already-established members, we ran the entire catalog through the Python program and graphed the result (see **Fig. 6**). A large spike of stars with very low probability values coincident with a broad distribution at high probability values suggests that a low probability value is a strong indicator that a star is excluded from cluster membership. However, a high probability value is not a definitive measure of cluster membership. This result was expected, as Messier 35 is located in the Galactic plane and corotating with the rest of the Galactic thin disk stars. Our analysis also reinforced that the equation produced an expected scattering of probability values in accordance with the original catalog. Having confirmed that our equation adequately fulfilled our purposes, we then proceeded to use it on our list of WD cluster candidates.

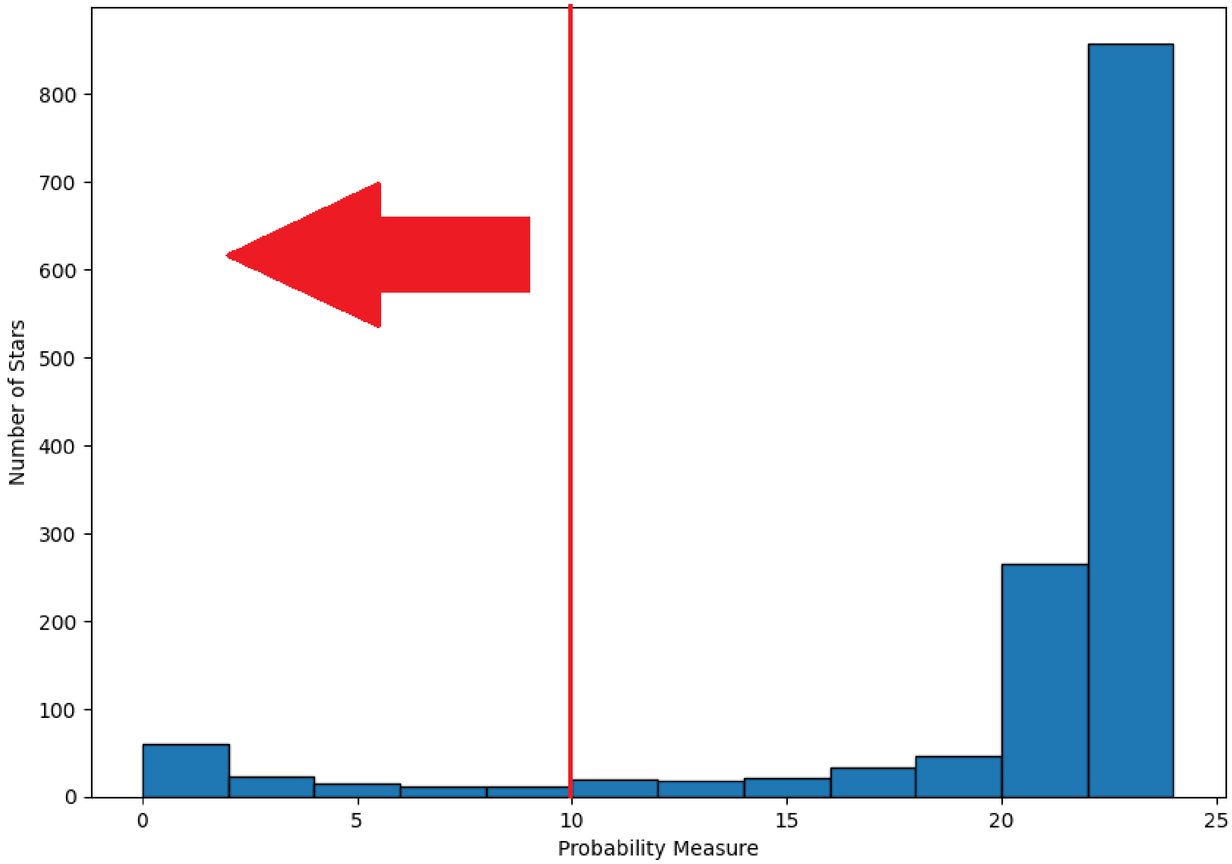


Fig. 5. A histogram showing the distribution of probability values for stars shown to be Messier 35 cluster members in the DANCe catalog (Buoy et al. 2015). Among the member candidates, stars with probability measures less than or equal to 10 are excluded from cluster membership at the 90th percentile confidence level (annotated above by the area left of the red line). As expected, a majority of already-established members from the DANCe study also appeared to be members in our analysis.

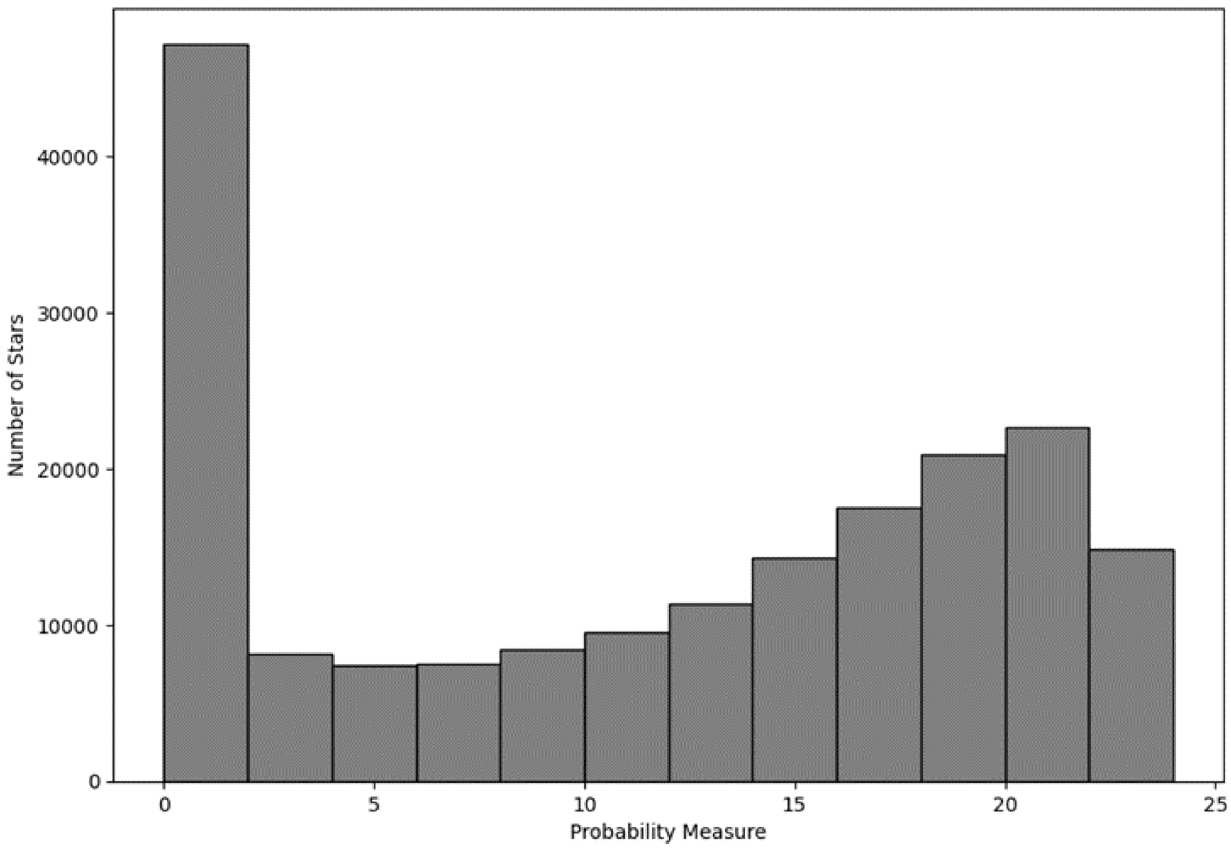


Fig. 6. A histogram showing the probability measures of all stars in the DANCe Messier 35 catalog (Buoy et el. 2015). This follows the expected trend in that a majority of stars in the area surrounding Messier 35 prove to be field stars, with a smaller collection of certain members and a scattering of in-between values.

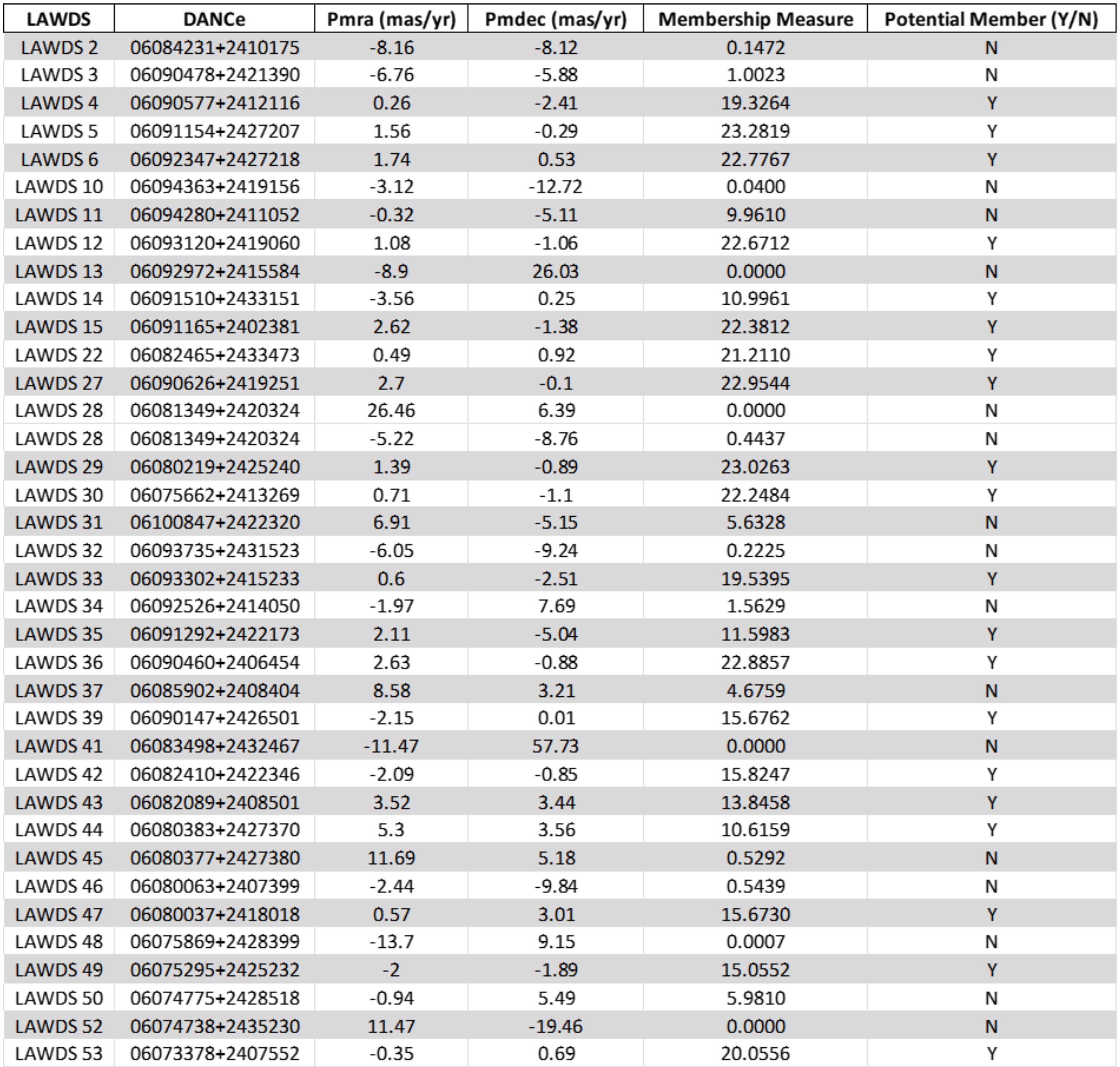
Once we had queried the catalog for proper motions for our WD candidates, we input the resulting data from Excel into our Python program. The program then utilized the NumPy and Pandas libraries to store the data in a structure known as a dataframe. The probability calculation was then applied to each row in the dataframe and resulting probabilities added to their respective entries. The program then exported the analysis results to Excel for closer inspection and future reference. We also made use of the dataframe structure within Python and were able to plot a variety of helpful charts while analyzing our results, including proper motion diagrams, histograms, and scatter plots. All of these tools allowed us to both visualize and establish confidence in our analysis results.

Our calculations showed that fifteen of the thirty-eight potential WD member candidates of Messier 35 had very low membership probabilities and could be reasonably excluded from further consideration as cluster candidates. Twelve candidates appeared to be likely members, with the remaining nine candidates necessitating further analysis (see **Fig. 7**).

Notes on Individual Objects

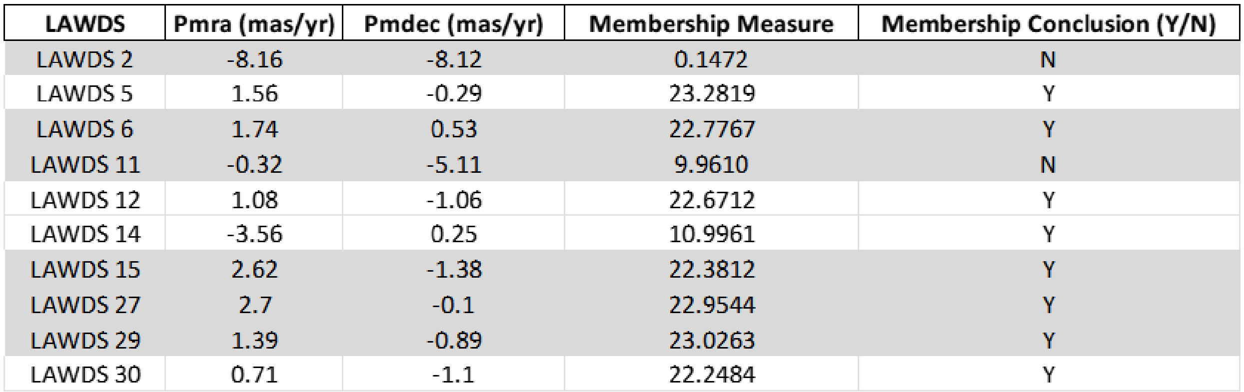
LAWDS 4

LAWDS 4 was included in the group of potential members, with our probability measurement indicating a strong consistency with cluster membership. As a DB WD with a helium-dominated atmosphere, LAWDS 4's potential membership in Messier 35 opens the door to further analysis of its initial-final mass relationship. Establishing LAWDS 4 as a cluster member indicates that Messier 35 is potentially the youngest open star cluster with a DB WD member (Williams et al. 2009). As previously indicated by Williams et al., LAWDS 4's distance is consistent with Messier 35 cluster membership. Additionally, our work shows its proper motion to also be consistent with cluster membership. Messier 35 membership would necessitate a high progenitor mass for the WD. However, because its mass is lower than the traditional initial-final mass relation, it seems likely that LAWDS 4 is the result of a binary formation process.



LAWDS	DANCe	Pmra (mas/yr)	Pmdec (mas/yr)	Membership Measure	Potential Member (Y/N)
LAWDS 2	06084231+2410175	-8.16	-8.12	0.1472	N
LAWDS 3	06090478+2421390	-6.76	-5.88	1.0023	N
LAWDS 4	06090577+2412116	0.26	-2.41	19.3264	Y
LAWDS 5	06091154+2427207	1.56	-0.29	23.2819	Y
LAWDS 6	06092347+2427218	1.74	0.53	22.7767	Y
LAWDS 10	06094363+2419156	-3.12	-12.72	0.0400	N
LAWDS 11	06094280+2411052	-0.32	-5.11	9.9610	N
LAWDS 12	06093120+2419060	1.08	-1.06	22.6712	Y
LAWDS 13	06092972+2415584	-8.9	26.03	0.0000	N
LAWDS 14	06091510+2433151	-3.56	0.25	10.9961	Y
LAWDS 15	06091165+2402381	2.62	-1.38	22.3812	Y
LAWDS 22	06082465+2433473	0.49	0.92	21.2110	Y
LAWDS 27	06090626+2419251	2.7	-0.1	22.9544	Y
LAWDS 28	06081349+2420324	26.46	6.39	0.0000	N
LAWDS 28	06081349+2420324	-5.22	-8.76	0.4437	N
LAWDS 29	06080219+2425240	1.39	-0.89	23.0263	Y
LAWDS 30	06075662+2413269	0.71	-1.1	22.2484	Y
LAWDS 31	06100847+2422320	6.91	-5.15	5.6328	N
LAWDS 32	06093735+2431523	-6.05	-9.24	0.2225	N
LAWDS 33	06093302+2415233	0.6	-2.51	19.5395	Y
LAWDS 34	06092526+2414050	-1.97	7.69	1.5629	N
LAWDS 35	06091292+2422173	2.11	-5.04	11.5983	Y
LAWDS 36	06090460+2406454	2.63	-0.88	22.8857	Y
LAWDS 37	06085902+2408404	8.58	3.21	4.6759	N
LAWDS 39	06090147+2426501	-2.15	0.01	15.6762	Y
LAWDS 41	06083498+2432467	-11.47	57.73	0.0000	N
LAWDS 42	06082410+2422346	-2.09	-0.85	15.8247	Y
LAWDS 43	06082089+2408501	3.52	3.44	13.8458	Y
LAWDS 44	06080383+2427370	5.3	3.56	10.6159	Y
LAWDS 45	06080377+2427380	11.69	5.18	0.5292	N
LAWDS 46	06080063+2407399	-2.44	-9.84	0.5439	N
LAWDS 47	06080037+2418018	0.57	3.01	15.6730	Y
LAWDS 48	06075869+2428399	-13.7	9.15	0.0007	N
LAWDS 49	06075295+2425232	-2	-1.89	15.0552	Y
LAWDS 50	06074775+2428518	-0.94	5.49	5.9810	N
LAWDS 52	06074738+2435230	11.47	-19.46	0.0000	N
LAWDS 53	06073378+2407552	-0.35	0.69	20.0556	Y

Fig. 7. A table of analysis results from our membership probability calculation process. Our WD member candidates are identified by name and DANCe identification number with their respective proper motions listed. The listed membership measure value is not a direct probability in percentage form, but rather a relative measure of membership likelihood based on analysis of previous datasets. The "Potential Member (Y/N)" column indicates our proposed interpretation of each analysis, with candidates marked "N" excluded from cluster membership at the 90th percentile level. We note the unusual case of LAWDS 28, which has two entries in the table coinciding with two catalog entries it matched with in our analysis. Further discussion of this item continues below.



LAWDS	Pmra (mas/yr)	Pmdec (mas/yr)	Membership Measure	Membership Conclusion (Y/N)
LAWDS 2	-8.16	-8.12	0.1472	N
LAWDS 5	1.56	-0.29	23.2819	Y
LAWDS 6	1.74	0.53	22.7767	Y
LAWDS 11	-0.32	-5.11	9.9610	N
LAWDS 12	1.08	-1.06	22.6712	Y
LAWDS 14	-3.56	0.25	10.9961	Y
LAWDS 15	2.62	-1.38	22.3812	Y
LAWDS 27	2.7	-0.1	22.9544	Y
LAWDS 29	1.39	-0.89	23.0263	Y
LAWDS 30	0.71	-1.1	22.2484	Y

Fig. 8. A table of analysis data comparing proposed members from Williams et al. (2009) to our current analysis results.

LAWDS 28

Unfortunately, the hot DQ white dwarf LAWDS 28 remains stubbornly ambiguous. One reason behind this result may be poor data conditions beyond the control of the original survey. LAWDS 28 matched with two entries in the catalog, showing that it may have been difficult to observe and capture proper measurements for. One of the matching entries is a relatively high proper motion object. This entry does not seem to align with LAWDS 28, as its proper motion in right ascension is too large. Through a comparison of images taken of LAWDS 28 at the Canada-France-Hawaii Telescope in 1999 and the Cerro Tololo Interamerican Observatory's Blanco Telescope in 2019, we determined that LAWDS 28 did not move during that time by more than 0.03 arcsec in right ascension. However, disqualification of this large proper motion entry also leads us to question the validity of the smaller proper motion entry.

Through image analysis we realized that the WD LAWDS 28 is blended with a fainter, very red extended object (potentially a galaxy) in most images, which could be the reason for the inaccurate proper motions in the DANCe catalog. While we could not find evidence to support LAWDS 28's membership in Messier 35, further data collection and analysis is necessary in order to rule out its possible cluster membership. We hope future photometric data of this hot DQ WD will provide further insight into its membership status.

Statement of Limitations

Our research topic is relatively narrow, and not many probability-based studies have been done on this particular topic. As a result, more research on the cluster membership of WDs within Messier 35 is undoubtedly necessary. We acknowledge that our research was conducted over a limited amount of time, with measurements that unavoidably contain some amount of error. However, we completed our goal of researching our subject as comprehensively as possible in the time that we had.

The data we collected was exclusively numerical, astronomical data. Our study did not require approval from the IRB, as it did not relate in any way to human or animal subjects. Additionally, there were no human-subject-related ethical concerns posed by our study. Our ethical aims as we conducted our research were to adhere to the standard University research protocols and present accurate, unbiased analysis and representation of the numerical data.

Conclusion

Our research contributes to the existing astronomical data concerning WDs, specifically those determined to be members of Messier 35. Our review of existing literature reflects the vital nature of white dwarf star analysis. Establishing cluster membership of white dwarfs provides broadly applicable insight concerning stellar aging, time, and mass evolution. Through the application of a two-dimensional Gaussian model to existing proper motion and parallax data for LAWDS 28, LAWDS 4, and other potential WD Messier 35 members, we evaluated their respective membership probabilities. The resulting probabilities highlight twelve potential cluster members, including LAWDS 4. Additionally, we emphasize the necessity of further research and analysis of better photometric data of other candidates (including LAWDS 28). By estimating the membership probabilities of these WDs and exploring the implications of those memberships, we establish the importance of future research concerning these WDs.

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